JULY, 1901.

THE MICHELSON ECHELON DIFFRACTION GRATING.

Issued from the Optical Workshop of

ADAM HILGER,

75a, CAMDEN ROAD,

LONDON, N.W.

THE MICHELSON ECHELON DIFFRACTION GRATING.

The Echelon Diffraction Grating was the result of endeavours on the part of its inventor to obtain means of higher resolution of the spectrum than were at the time available. Trains of prisms have, of course, long been superseded where high resolving power was desired, recourse being had to ruled diffraction gratings, of which those ruled on Prof. Rowland's engine hold a monopoly of the field.

The resolving power of a spectroscope must be clearly distinguished from its dispersion. The latter is a measure of the angular separation produced between two given monochromatic rays, while the former is a measure of the number of monochromatic rays which it is possible to distinguish as separate between two given monochromatic rays, and is the really important criterion of the power of the instrument. For it must be remembered that the image produced by a telescope—the termination of every spectroscope—is not the indefinitely thin line given by the construction of geometrical optics, but one of appreciable breadth, shading off to darkness on each side. The smaller the aperture of the telescope (as limited either by the diameter of its own object glass or by the width of the beam of light from the prism or grating system) the broader do the lines become. If then the wave-lengths of two rays differ by so little that these lines overlap with a particular aperture, an increase of aperture alone, provided it be sufficiently great, is sufficient to separate these rays.

The resolving power of a grating can then be indefinitely increased by ruling lines over a wider area without alteration in the number of lines to the millimetre, for the effective aperture being increased the spectral lines become finer and hence separable. The resolving power is in fact proportional to the length of ruling—that is, to the travel of the grating plate during the cutting.

Unfortunately the difficulties of cutting accurate gratings increase very rapidly as the length of ruling increases, so that gratings with 5 or 6-inch ruled surfaces are the biggest that can be obtained, and even these are very difficult to get.

There is another way of getting higher resolving power. The resolving power is given by the product, $m n$, where m is the order of spectrum observed and n the number of elements of which the grating is composed—the number of divisions, that is, in the case of ruled gratings.

With the ordinary form of grating the intensity rapidly diminishes as the order is raised; but if a grating could be constructed to throw most of the light in one of the high orders the desired object would be attained.

The device which was first suggested by Michelson to obtain this result was as follows.

Let a number of plates of silvered glass of equal thickness, t, be superposed, each projecting beyond the last by an equal amount in such a manner as to form a series of steps, as shown in the diagram.

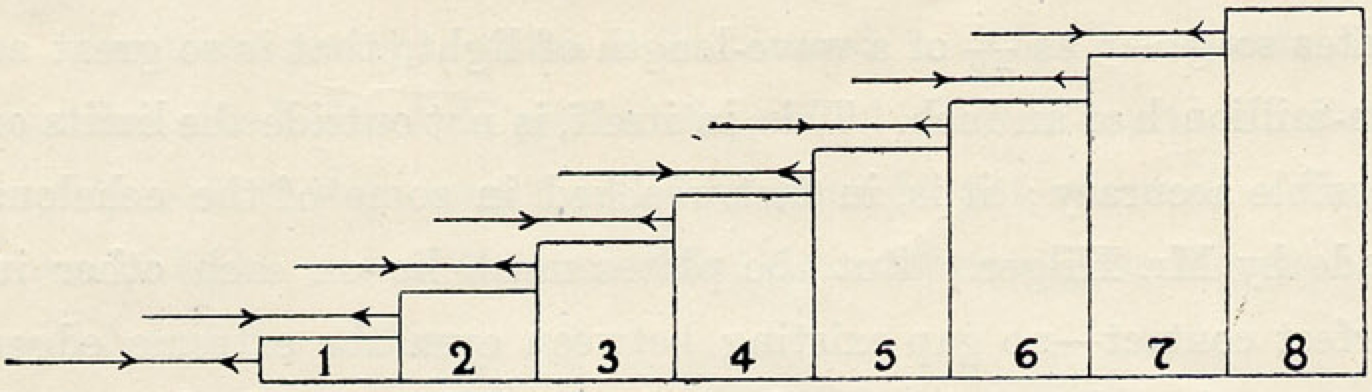

FIG. 1. REFLECTION ECHELON.

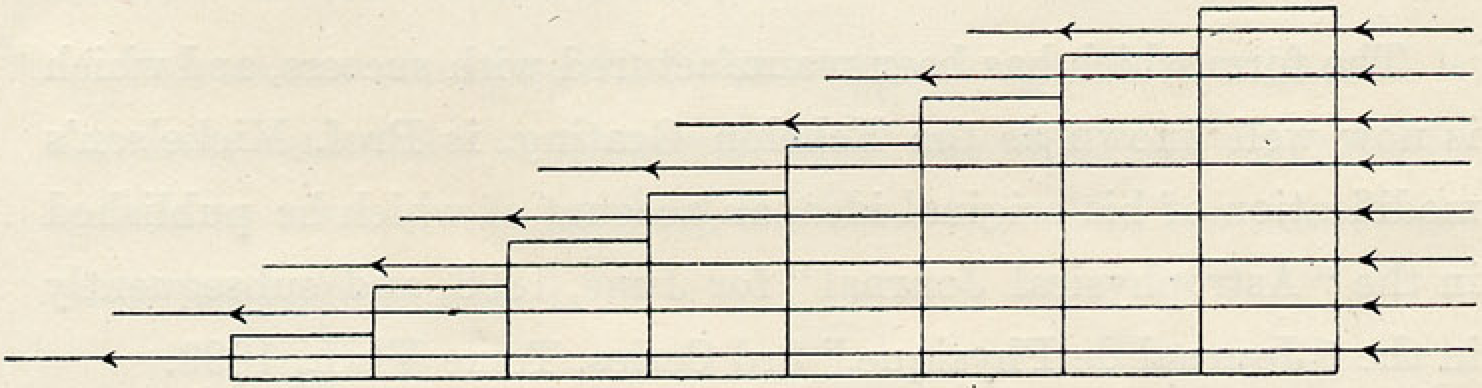

FIG. 2. TRANSMISSION ECHELON.

If then a beam of "parallel" light be incident in the direction of the arrows, it will be reflected from the successive steps, and the reflected beam will consist of a number of beams reflected from the 1st, 2nd, 3rd, etc. plates, whose retardations behind that reflected from the first plate will be 0, t, $2t$, $3t$, etc. They will in fact be in the requisite relation to produce, when brought to a focus by a lens, diffraction spectra of the light source.

If t be taken equal to 10 mm. the orders of spectra observed would be in the neighbourhood of the 34,000th, so that five plates would give a resolving power greater than that of a 10-inch grating in the first order.

It is, however, unnecessary to go into the analytical expressions for this form as it has not yet been found possible to manufacture it. The accuracy required in the parallelism of the plates in order for the grating to perform in accordance with the theoretical expressions of its capabilities would be such, for one of ten plates, that no differences in thickness should exist among the different plates so great as $\frac{1}{20}$ of a wave-length of light, that is so great as one-millionth of an inch. This, in itself, is not outside the limits of possible accuracy; it is, in fact, reached in some of the echelons made by Mr. Hilger. But the plates must lie on each other in perfect contact—no gap existing between even the extreme edges of successive plates of as much as the quantity stated above, and this is the difficulty which has not yet been surmounted.

The form which has been manufactured with success, and which is now well known as the Echelon Grating, is Prof. Michelson's modification of his original idea, an account of which he published in the "Astrophysical Journal" for June, 1898, and subsequently in the "Journal de Physique," 3rd Series, Tome VIII., 1899.

It is from these sources that the following analytical investigations are taken.

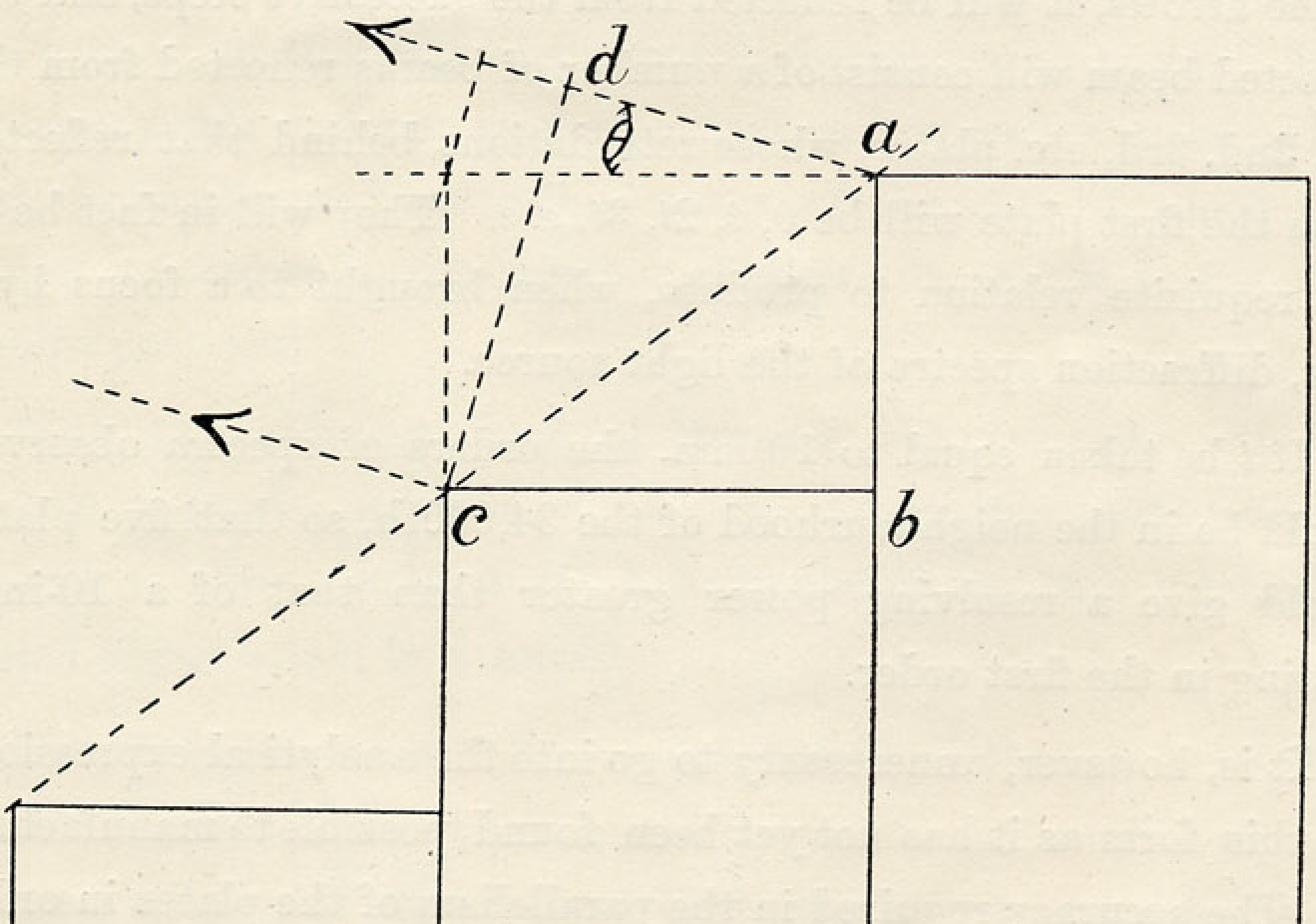

FIG. 3.

The arrangement, which is illustrated in Fig. 2, consists, like that described above, of plates of parallel glass of equal thickness superposed in step form; but the glass is not silvered, and the beam of light passes through the mass emerging at the exposed surfaces of the plates.

The beam of parallel light is produced by a collimator, and the light is received in a telescope.

The beams emerging from the various apertures will then be relatively retarded by successively equal amounts, and will produce spectra by interference in much the same way as in the case of the reflection form in Fig. 1.

THEORY OF THE ECHELON.

Suppose a beam of "parallel" homogeneous light be incident normal to the plates.

Let t = thickness of the plates ($b\ c$, Fig. 3).

s = width of the steps ($a\ b$, Fig. 3).

λ = wave-length of the light under consideration.

μ = refractive index of the glass for light of wave-length λ.

m = retardation in terms of λ between rays emerging at an angle of diffraction θ from corresponding points (such as a and c, Fig. 3) of successive elements.

n = number of elements.

Then (*see* Fig. 3)—

$$m\lambda = \mu t - a d$$
$$= \mu t - t \cos\theta + s \sin\theta.$$

Since the values of θ which are of interest are very small, this may be written—

$$(1) \quad m\lambda = (\mu - 1)t + \theta s$$

Differentiating with respect to λ, m constant, and re-arranging the terms—

$$\frac{d\theta_1}{d\lambda} = \frac{1}{s}\left(m - t\,\frac{d\,\mu}{d\,\lambda}\right)$$

or, substituting for m its approximate value $(\mu - 1)\,\frac{t}{\lambda}$,

$$(2)\quad \frac{d\theta_1}{d\lambda} = \frac{t}{s\,\lambda}\left[(\mu - 1) - \lambda\,\frac{d\,\mu}{d\,\lambda}\right] \text{ - - - - - - } (\textit{dispersion})$$

$$= \frac{b\,t}{s\,\lambda}$$

The value of the co-efficient b can be found for any wave-length from the optical constants for the glass used (it is between 0·5 and 1·0 for most glasses), and the equation (2) will then give the angular dispersion $d\theta_1$ between two homogeneous radiations whose wave lengths differ by a small amount $d\,\lambda$.

Differentiating (1) with respect to m, λ constant, and re-arranging the terms—

$$\frac{d\theta}{dm} = \frac{\lambda}{s}$$

Putting $dm = 1$, and representing the corresponding change in θ by $d\theta_2$, then—

$$(3)\quad d\theta_2 = \frac{\lambda}{s} \text{ - - - - - - } (\textit{separation of spectra}).$$

Equation (3) gives the angular separation between two successive orders of spectra.

Let $d\theta_3$ represent the angular limit of resolution of the echelon, *i.e.*, the angular separation of two spectral lines when they are just distinguishable as separate in the eyepiece of the telescope (*see* p. 1). By the well-known rule (*see* Lord Rayleigh on "Wave Theory," *Enc. Brit.*)

$$(4)\quad d\theta_3 = \frac{\lambda}{\text{effective aperture of telescope object-glass.}}$$

$$= \frac{\lambda}{n\,s} = \frac{d\theta_2}{n}$$

Let $d\lambda_3$ be the difference of wave-length corresponding to the angle $d\theta_3$.

Then from (2) $\frac{d\theta_3}{d\lambda_3} = \frac{b\,t}{s\,\lambda}$

or, substituting $\frac{\lambda}{n\,s}$ for $d\theta_3$ and rearranging terms,

$$(4a) \quad \frac{d\lambda_3}{\lambda} = \frac{\lambda}{b\,n\,t} \text{ - - - - - - } (\textit{limit of resolution})$$

$d\lambda_3$ in the above represents the difference of wave-length corresponding to the angle $d\theta_3$, *i.e.*, the difference of wave-lengths of the closest separable pair of homogeneous rays; and the ratio $\frac{\lambda}{d\lambda_3}$ is hence the resolving power of the echelon. This is, it will be seen, proportional to the total thickness of glass traversed, and for a given wave-length is independent of the thickness of plates or width of step.

The brilliancy of any line seen with the echelon depends not only on the intrinsic intensity of the radiation, but also on the angle of diffraction, θ. The co-efficient which depends on θ, and which may be called the Intensity of Illumination, is

$$(5) \quad I = \left[\frac{\sin \pi \frac{s}{\lambda} \theta}{\pi \frac{s}{\lambda} \theta} \right]^2$$

APPLICATION OF THE FOREGOING EXPRESSIONS TO AN ACTUAL CASE.

The echelon whose performance will be investigated is one made by Mr. Hilger for Dr. Hauswaldt of Magdeburg; the quantities being as follows:—

Thickness of plates, t = about 10 *mm.*

Width of steps, $s = 1$ *mm.*

Refractive index of the glass :—

$$\mu_C = 1{\cdot}5706$$
$$\mu_D = 1{\cdot}5746$$
$$\mu_F = 1{\cdot}5845$$
$$\mu_{G1} = 1{\cdot}5927$$

Whence $\frac{d\mu}{d\lambda}$ for W. L. 5890 (D) $= -7{\cdot}19 \times 10^{-6}$, λ being in 10^{-10} metres.

Number of elements, $n = 21$.

(It is to be noted that when a clear aperture of 1 *mm.* is left beyond the width of the largest plate the number of apertures operative in the formation of the spectrum is one more than the number of plates. It is this number of apertures which is denoted by *n*.)

THE DISPERSION.

From (2) p. 6.

$$\frac{d\theta_1}{d\lambda} = \frac{b\,t}{s\,\lambda}, \text{ where } b = (\mu - 1) - \lambda\,\frac{d\mu}{d\lambda}.$$

Substituting for μ, λ, and $\frac{d\mu}{d\lambda}$ their values for D,

$$b_D = 0{\cdot}575 + 0{\cdot}042$$
$$= 0{\cdot}617$$

Hence $\frac{d\theta_1}{d\lambda} = {\cdot}00105$ in the region of D.

Putting $d\lambda = 0{\cdot}1$, $d\theta_1 = {\cdot}000105$ radians.

$\doteqdot 0{\cdot}37$ minutes of arc.

this being, then, the dispersion between two rays which differ in wave-length by $\frac{1}{10}$th of an Ångström unit.

THE SEPARATION OF SUCCESSIVE ORDERS OF SPECTRA.

From (3) p. 6, the angular separation of successive orders, denoted by $d\theta_2, = \frac{\lambda}{s}$

Hence for D—

$$d\theta_2 = 5{\cdot}89 \times 10^{-4} \text{ radians.}$$
$$\doteq 2 \text{ minutes of arc.}$$

The successive orders are, then, very close together, and an auxiliary spectroscope is necessary for preliminary analysis of the light; only a very restricted portion of the spectrum under observation being thrown on the slit of the echelon spectroscope.

The Angular Limit of Resolution.

From (4) p. 6 the angular separation of two lines which are just distinguishable as separate, denoted by $d\theta_3 = \frac{d\theta_2}{n}$

Hence for D—

$$d\theta_3 = \frac{d\theta_2}{21} = \text{(for D) } 0{\cdot}095 \text{ minutes of arc.}$$

From (4*a*) p. 7 the difference of wave-length necessary between two homogeneous radiations in order to distinguish them as separate, denoted by $d\,\lambda_3$

$$= \frac{\lambda^2}{b\,n\,t} = \text{(for D) } \frac{5890^2}{0{\cdot}617 \times 21 \times 10^8} \text{ Ångström units.}$$
$$= 0{\cdot}027 \text{ Ångström units.}$$

It must be noted that one can determine the *displacement* of a single ray more readily than one can separate the components of a close doublet. In fact, a shift of a line of as little as $\frac{1}{10}$th the angular limit of resolution can be detected. (*See* " Le Phénomène de Zeeman"—A. Cotton, p. 15.)

THE EFFECT OF ALTERATION OF DIMENSIONS OF THE ECHELON.

These may be summarised as follows (*see* (2), (3), (4), (4*a*), pp. 6 and 7):—

Increase in thickness of plates increases the dispersion, and therefore the amount of detail distinguishable. The separation of spectra remains the same.

Increase of width of step decreases the separation of successive spectra; but the angular limit of resolution is reduced also, the amount of detail distinguishable remaining the same.

Increase of number of plates does not alter dispersion or separation of spectra; but it reduces the angular limit of resolution, the amount of detail distinguishable being thereby increased. In addition to this the amount of light is increased.

THE INTENSITY CURVE.

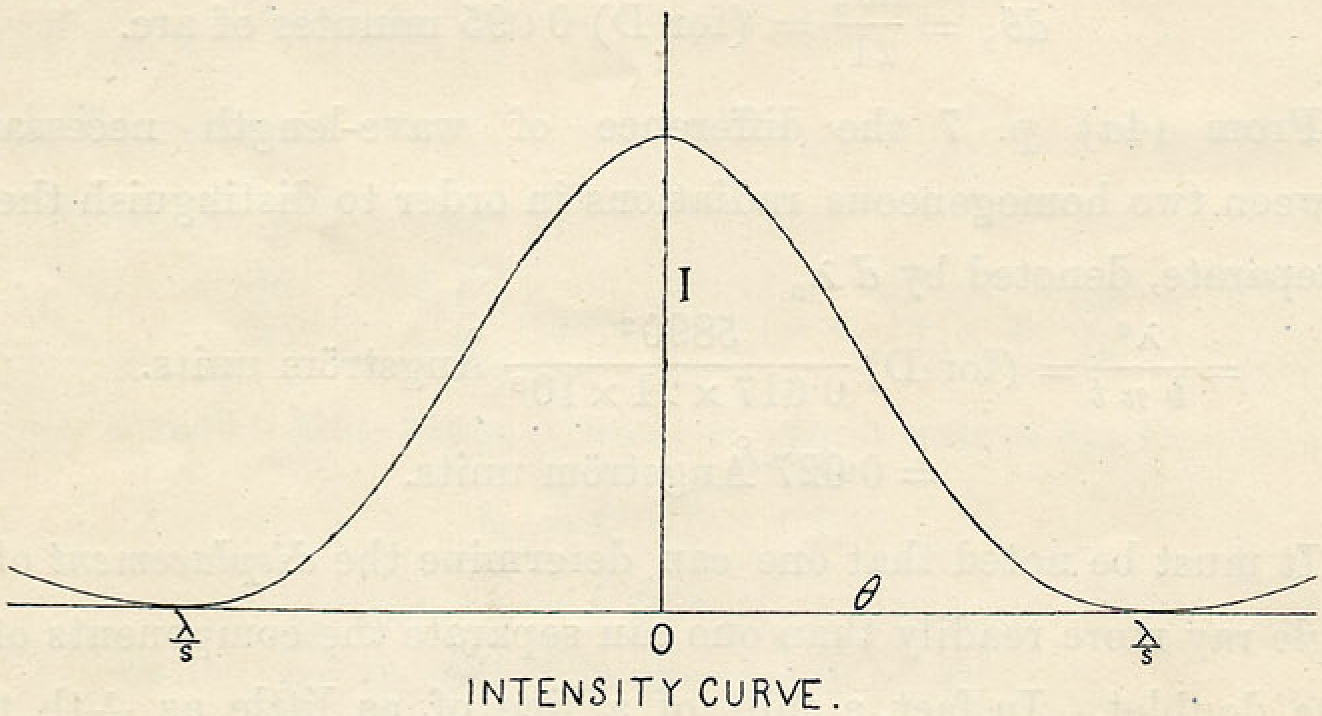

FIG. 4.

The curve in Fig. 4 has been plotted from the expression for the intensity in (5) p. 7.

It will be seen that I = 0 when $\theta = \frac{\lambda}{s}$. Now $\frac{\lambda}{s}$ is the angular separation of successive orders of the same line, hence if any spectral line be formed at $\theta = 0$, the two orders next it on each side are absent, and practically all the light is concentrated in the one line.

This fact should be well noted, as by a slight rotation of the echelon it is found that any desired line can be got into this position, and the property above mentioned is one of great importance in the practical manipulation of the echelon.

MOUNTING, Etc.

Mr. Hilger, having made echelons for many of the principal Universities of Europe during the last two years, is in a position to propose with confidence the following dimensions, as those which have been found most generally satisfactory, and which he always supplies when no contrary instructions are given in ordering—

Thickness of plates	10 mm.
Width of step	1 mm.

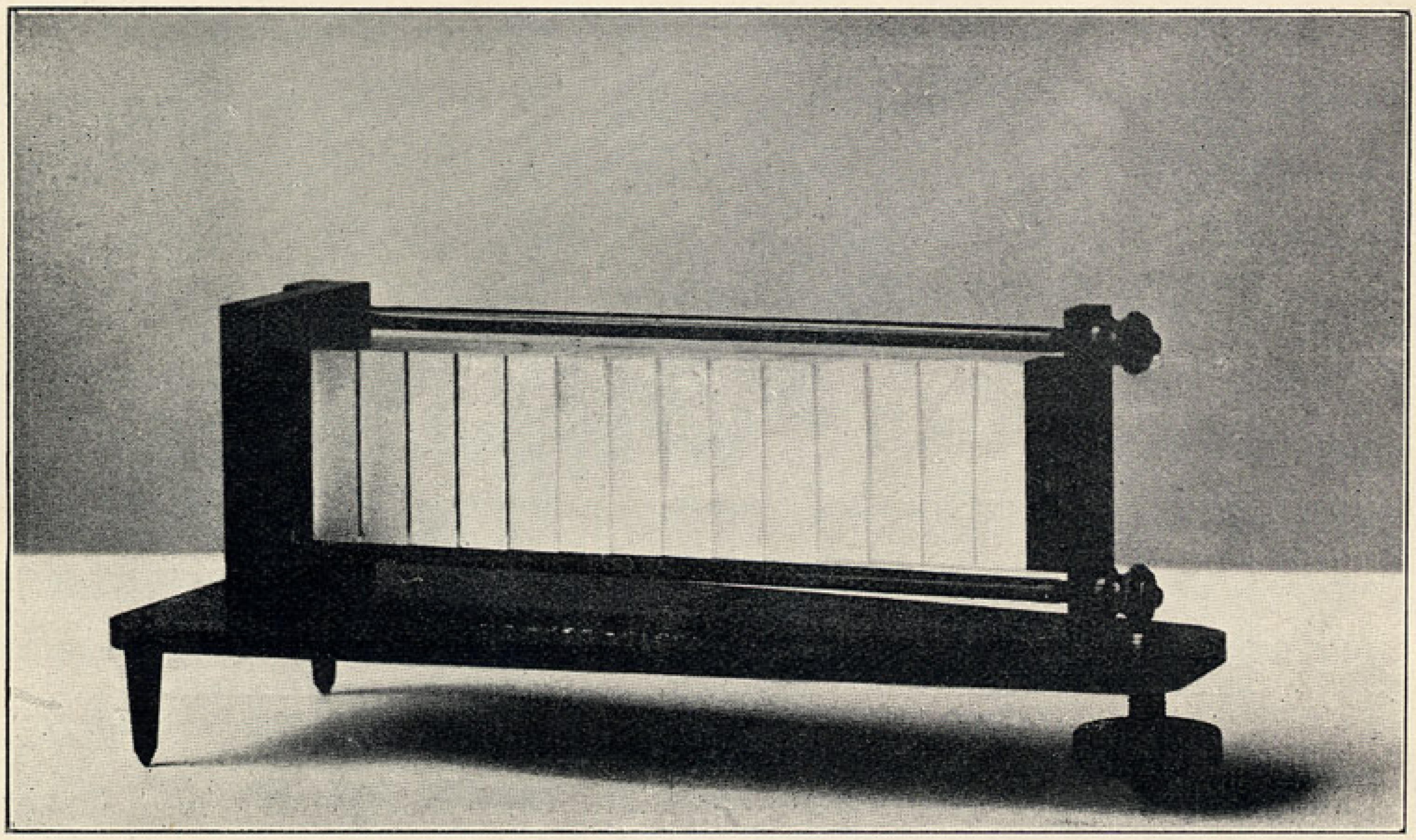

Fig. 5.

The mounting adopted varies somewhat according to the number of plates. Fig. 5 shows the mounting of a small echelon, one of fourteen plates constructed for the University of Vienna. The largest plate of glass rests on an optically worked metal plate in which the aperture is cut, and the plates are clamped by steel rods with millhead nuts. The plates are cleaned with the most scrupulous care, and when clamped in the above manner they make optical contact over the greater part of their surfaces.

By this means the great loss of light noticed by Prof. Michelson (who did not clamp the plates of his echelons) is obviated to a surprising degree, and the more recently made echelons are remarkable for their brilliancy; while experience shows that, contrary to expectation, judicious clamping does not cause the slightest loss of definition. The smaller sizes of echelon can be adapted to almost any form of spectroscope, but owing to its peculiarity of being always used at direct vision, a special form becomes more convenient.

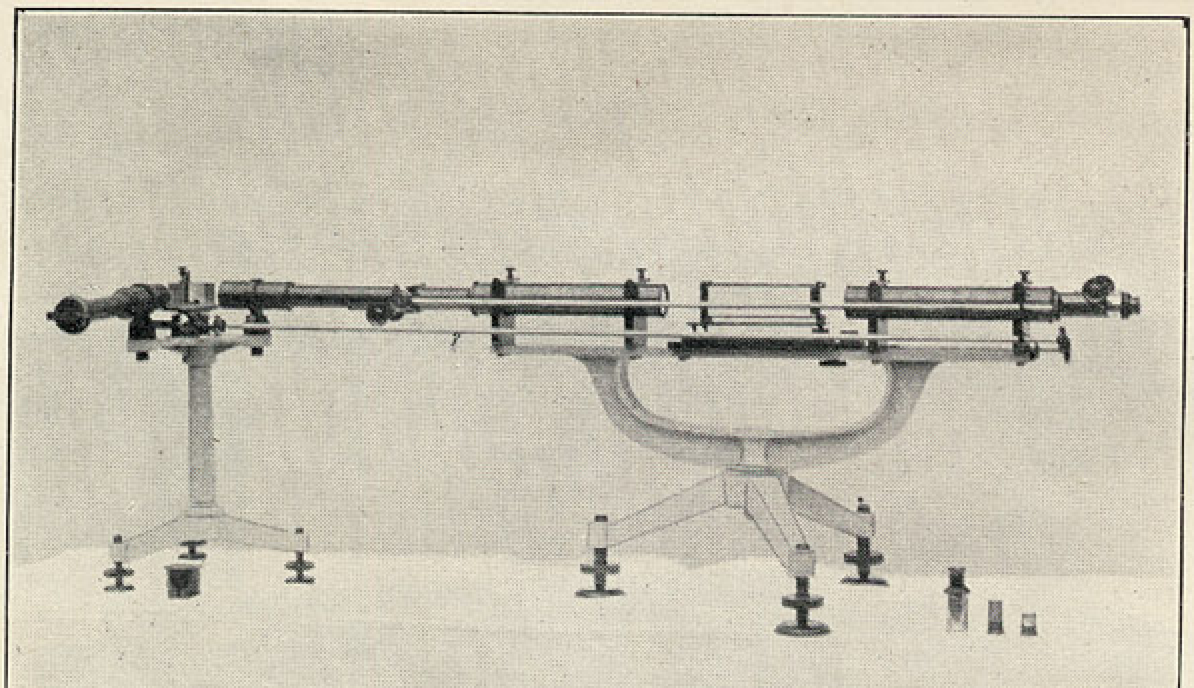

FIG. 6.

Such an echelon spectroscope, designed for a twenty plate or larger echelon, is shown in Fig. 6 in conjunction with a special form of auxiliary spectroscope (known as the "Constant Deviation Spectroscope") which presents many advantages for the purpose;

the chief of which is that neither collimator nor telescope is ever moved. Arrangements are provided by means of rods and bevel gear for the various adjustments that have to be made, such as the passing through the spectrum by rotation of the prism of the auxiliary spectroscope, the opening and shutting of the slit, the rotation of the echelon to get the line under observation into any desired position, and the moving of the echelon out of the field of view.

HINTS ON THE USE OF THE ECHELON.

It is well worth while spending a little time in carefully setting up the apparatus in the first instance, care being specially taken that the whole of the echelon is operative. To do this one should proceed as follows:—

Set the echelon and auxiliary spectroscopes up end to end as shown in Fig. 6, with the axis of each horizontal and at exactly the same height from the table. The eye-piece should, of course, be removed from the auxiliary spectroscope, and the distance from the end of its telescope tube to the focus of the telescope object glass being carefully measured beforehand, the auxiliary spectroscope should be put at such a distance from the slit of the echelon spectroscope as to bring the spectral lines to a focus on the slit. Set the telescope of the auxiliary and the collimator of the echelon spectroscope in one line as nearly as can be judged by the eye. Now put a sodium flame in front of the auxiliary slit, and turn the prism of the auxiliary spectroscope until D comes on to the "echelon" slit. Then on looking through the echelon spectroscope with the eye-piece removed the object glasses should appear equally illuminated all over, except perhaps two slight black strips right and left, which if present should be perfectly symmetrical. When this is the case (and the setting is not right until it is so) the apparatus is ready.

If now the slit of auxiliary spectroscope be closed to a convenient width and that of the echelon spectroscope be opened wide the spectrum of any light placed in front of the slit of the auxiliary spectroscope will be seen in the eye-piece of the viewing telescope. Any particular line that it is desired to examine must then be got on to the fixed jaw of the slit, the slit closed to exclude all other lines, and the echelon introduced. Supposing the radiation to be homogeneous within the limit of resolution of the echelon, the following possible conditions may be observed in the eye-piece:—

1. There may be one bright single line visible, together with a series of very faint lines gradually diminishing in intensity as the distance from the central line is increased. This is the condition in which the two neighbouring orders to right and left are absent (*see* p. 10).

2. There may be two lines of equal intensity, which are successive orders of one and the same line, also with a series of very faint lines diminishing in intensity towards the edge of the field.

3. There may be any number of conditions intermediate between these two, with two lines visible but of unequal brilliancy, and the series of very faint lines as before. The series of lines to right and left will not be found to interfere with the use of the instrument in the slightest degree—there are two points right and left which are *always dark* and which enclose a space equal to twice the separation of successive orders (*see* p. 10). It is on this region that the attention should be concentrated.

By a slight rotation of the echelon about a vertical axis any of the conditions above-mentioned can be attained—indeed, one can pass a large number of orders through the field one after the other. There is one position of the echelon, however, at which, during rotation, the lines reverse the direction of their motion in much the same way as the lines in a prism spectrum when the prism is rotated through the position of minimum deviation. This is when the echelon plates are normal to the incident light, and this position, or near it, is the best to work at.

USES OF THE ECHELON.

As will be seen by reference to p. 9, the range of spectrum which can be observed at one time is very small; but with suitable means (*see* pp. 11 and 12) the passing from one line to another becomes very rapid and convenient. The purpose, then, for which the echelon is peculiarly suited is the analysis of minute details of structure or of variation of spectral lines. Such are, the determination of the effect of pressure on the nature of the radiations emitted by glowing gases etc., the effect of motion of the light source relative to the spectroscope, and above all the observation of the interesting phenomenon known as the "Zeeman Effect."

It may be of interest to note the magnitude of these various effects in one or two cases.

The difference between the velocities of two opposite points on the sun's equator is 4 kilometres per second, and this produces a relative shift of the spectral lines of 0·075 Å.U. As in the grating investigated above a shift of 0·003 Å.U. would be perceptible, it seems probable that the echelon may form a valuable tool in the hands of astronomers.

The displacements towards the red end of the spectrum, due to a pressure of eleven atmospheres in various lines, have been given by Humphreys (from experiments made in Rowland's Laboratory with the six-inch reflection grating; *see* "The Astrophysical Journal," Oct., 1897, p. 169). The greatest among those observed vary from 0·02 to 0·50 Å.U.

In the case of the Zeeman effect, with fields of about 30,000 C.G.S. units the separation of the components is of the order of 1·0 Å.U.

In conclusion it may be remarked that there is no difficulty in constructing echelons with plates considerably thicker than 10 mm. (Prof. Michelson in fact made them with 20 mm. plates) and thus increasing the resolving power, should this be desired.

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